# Conditional Truth Table, Validity Problem: Discussion

***Problem*:** Use truth tables to decide on the validity of the following argument.

1. $(P \rightarrow Q)$

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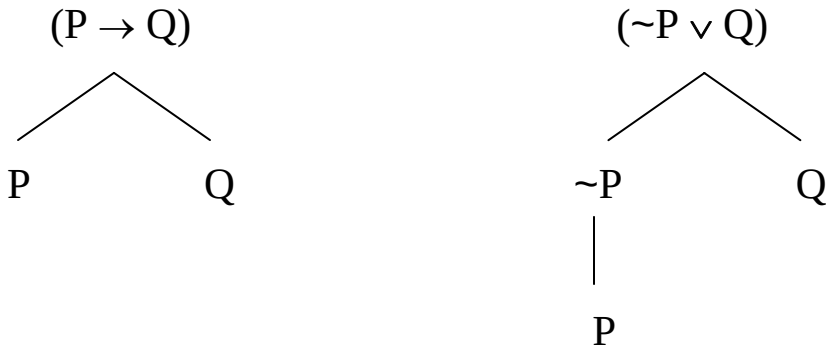
$\therefore (\sim P \vee Q)$

***Discussion***: A construction tree for each sentence shows which parts the truth table will require.




Since both sentences are built from atoms "P" and "Q," the truth table begins with these as well.

| **P** | **Q** |
|---|---|
| 1 | 1 |
| 1 | 0 |
| 0 | 1 |
| 0 | 0 |

We then add places in the table for the remaining sentences in the tree: "(P → Q)," "~P," and "(~P ∨ Q)".

| **P** | **Q** | **(P → Q)** | **~P** | **(~P ∨ Q)** |
|---|---|---|---|---|
| 1 | 1 | | | |
| 1 | 0 | | | |
| 0 | 1 | | | |
| 0 | 0 | | | |

"(P → Q)" follows the conditional rule.

| ● | ▲ | (● → ▲) |
|---|---|---|
| 1 | 1 | 1 |
| ⇨ **1** | **0** | **0** |
| 0 | 1 | 1 |
| 0 | 0 | 1 |

As line three of this rule shows, a conditional is only false when the antecedent is true, and the consequent is false. For conditional "(P → Q)" its antecedent is true and consequent false in the second valuation.

**Antecedent** **Consequent**

| **P** | **Q** | **(P → Q)** | **~P** | **(~P ∨ Q)** |
|---|---|---|---|---|
| 1 | 1 | | | |
| ⇨ 1 | 0 | 0 | | |
| 0 | 1 | | | |
| 0 | 0 | | | |

"(P → Q)" will be true in the other three valuations.

**Antecedent** **Consequent**

| **P** | **Q** | **(P → Q)** | **~P** | **(~P ∨ Q)** |
|---|---|---|---|---|
| 1 | 1 | **1** | | |
| 1 | 0 | **0** | | |
| 0 | 1 | **1** | | |
| 0 | 0 | **1** | | |

The negation "~P" follows the negation rule: when "P" is true, "~P" is false; and when "P" is false, "~P" is true.

| ● | ~● |
|---|---|
| 1 | 0 |
| 0 | 1 |

| **P** | **Q** | **(P → Q)** | **~P** | **(~P ∨ Q)** |
|---|---|---|---|---|
| 1 | 1 | 1 | 0 | |
| 1 | 0 | 0 | 0 | |
| 0 | 1 | 1 | 1 | |
| 0 | 0 | 1 | 1 | |

The disjunction "(~P ∨ Q)" follows the disjunction rule: a disjunction is only false when both parts – "~P" and "Q" – are false. That's valuation two.

| ● | ▲ | (● ∨ ▲) |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

| | | X | | X | |
|---|---|---|---|---|---|
| | **P** | **Q** | **(P → Q)** | **~P** | **(~P ∨ Q)** |
| | 1 | 1 | 1 | 0 | |
| ⇨ | 1 | **0** | 0 | **0** | **0** |
| | 0 | 1 | 1 | 1 | |
| | 0 | 0 | 1 | 1 | |

"(~P ∨ Q)" is true in the other three valuations.

| | X | | X | |
|---|---|---|---|---|
| **P** | **Q** | **(P → Q)** | **~P** | **(~P ∨ Q)** |
| 1 | 1 | 1 | 0 | **1** |
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | **1** |
| 0 | 0 | 1 | 1 | **1** |

To determine the validity of the argument, pick out the valuations that make all the premises true. Here, the only premise is "$(P \rightarrow Q)$," which is true in valuations 1, 3, and 4.

1. $(P \rightarrow Q)$

---

$\therefore$ $(\sim P \vee Q)$

| | | Premise | | Conclusion |
|---|---|---|---|---|
| **P** | **Q** | **$(P \rightarrow Q)$** | **$\sim P$** | **$(\sim P \vee Q)$** |
| 1 | 1 | **1** | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | **1** | 1 | 1 |
| 0 | 0 | **1** | 1 | 1 |

The conclusion is true in all those valuations. That makes the argument **valid**.

| | | Premise | | Conclusion |
|---|---|---|---|---|
| **P** | **Q** | **$(P \rightarrow Q)$** | **$\sim P$** | **$(\sim P \vee Q)$** |
| 1 | 1 | **1** | 0 | **1** |
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | **1** | 1 | **1** |
| 0 | 0 | **1** | 1 | **1** |

Verdict: **Valid**

**Note**: on a WebCT assignment, you would be asked to list the *sentences used* in building these truth tables. Reading across the top of the truth tables, we list the following sentences.

P
Q
$(P \rightarrow Q)$
$\sim P$
$(\sim P \vee Q)$